Q. **[Life] – Regarding the statement at the bottom of page 17 of Mr. Wiedmayer's rebuttal testimony addressing important considerations in curve-fitting, please provide all support and justification for the statement that "it is just as important to recognize which portions of the curve provide the most information about the different patterns for a group of assets." Further, specifically, what are the factors associated with "most information" in the noted statement and the ranking of the items of information from most important to least important. Finally, provide a copy of all authoritative sources that support the Company's response.**

A. The statement cited above actually reads "retirement pattern" instead of "different patterns" in Mr. Wiedmayer's testimony. The statement refers to the fact that for the Iowa Survivor curves, the majority of retirements occur in the middle portion of the curve. Thus, this portion of the curve contains the most information about the retirement pattern for a survivor curve. As presented in the Expert Rebuttal Evidence, Bulletin 125 supports this concept. Wolf and Fitch in *Depreciation Systems* also support this concept, as on page 47 they state that the "middle section is relatively straight and is the portion of the curve that often best characterizes the survivor curve."

The pages from Bulletin 125 are included as Attachment A and those from Wolf and Fitch are included as Attachment B.

Bulletin 125

THE COLLEGE

The Iowa State College of Agriculture and Mechanic Arts conducts work in five major fields:

- AGRICULTURE
- ENGINEERING
- HOME ECONOMICS
- INDUSTRIAL SCIENCE
- VETERINARY MEDICINE

The Graduate College conducts research and instruction in all these five fields.

Four-year, five-year, and six-year collegiate courses are offered in different divisions of the College. Non-collegiate courses are offered in agriculture. Summer sessions include graduate and collegiate work. Short courses are offered in the winter.

Extension courses are conducted at various points throughout the state.

Four special research institutes have been organized: the Agricultural and Engineering Experiment Stations, and the Veterinary and Industrial Science Research Laboratories.

Special announcements of the different branches of the work are supplied, free of charge, on application.

Address, THE REGISTRAR,
IOWA STATE COLLEGE,
Ames, Iowa.

...tical Analyses of Industrial Property Retirements

By

ROBLEY WINFREY, M.S.
Research Engineer

BULLETIN 125

...NG EXPERIMENT STATION

... COLLEGE
...ANIC ARTS

...CATION

No. 28

... and Mechanic Arts, Ames, Iowa.
... the special rate of postage pro-
..., authorized April 12, 1920

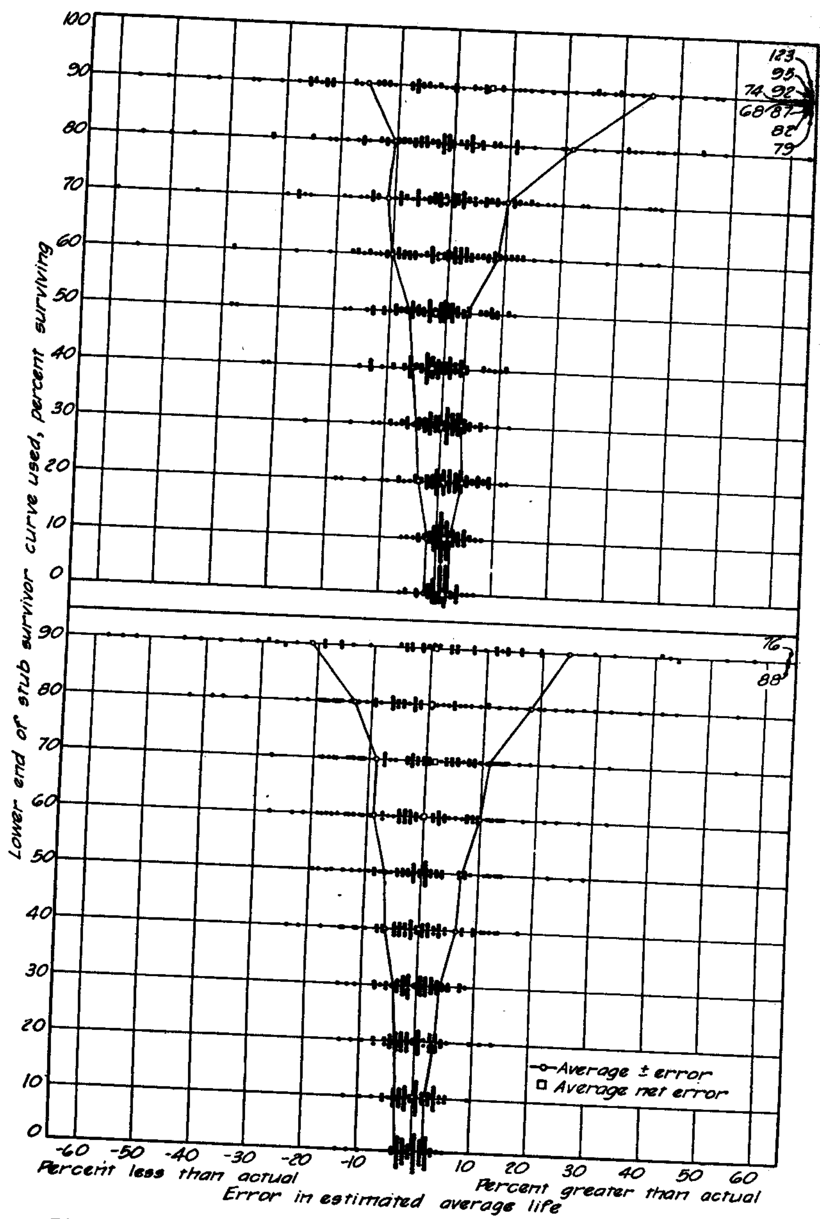

Fig. 30.—(Lower) Errors in estimating the probable average lives of the first 65 original curves by comparing stub curves of different lengths with the type survivor curves in the form shown in Fig. 29. (Upper) Same for curves 66 to 176.

and L_3, the S_1 and S_2, the R_2 and R_3, or other two adjacent curves in the same family. Another reason why the classifications are not the same is that the survivor curves for the high-modal curves are quite steep, and, therefore, these types when plotted as survivor curves appear to be about the same, except at the ends. The frequency curves emphasize the differences and are the better guides to classification.

The frequency curves are difficult to use in this method because of the scattering of the original data, which makes the location of the curve doubtful. In the case of original data well graduated, sets of the type frequency curves, plotted to definite average lives as is done in Fig. 29, were used successfully in a test similar to the two just described on a group selected from the first 65 curves. Ordinarily, this step is not warranted, for the probable average life estimated from the survivor curves is likely to be within the limits of error as controlled by the quantity and reliability of the original data.

The estimation of the probable average life of a group of units by comparing their survivor curve (completed curve or stub curve) with the type curves should not be done without the exercise of judgment in the interpretation of the original data. Any of the methods of constructing survivor curves frequently result in curves which do not exhibit regularity. An examination of the information from which the curves are calculated may show that the irregularity is produced by small groups, infrequent observations of the property, or the retirement of an unusually large number of units for a very special cause. Best practice in these instances is to smooth the data according to the path most likely to be established by regular observations on large numbers of the units and one in accordance with the most likely future rate of retirement.

When survivor curves are to be classified according to the 18 types and the probable average life determined, it is recommended that more weight be given to the middle portion of the survivor curve, say that between 80 and 20 percent surviving, than to the forepart or extreme lower end of the curve. This inner section is a result of greater numbers of retirements and also it covers the period of most likely normal operation of the property.

This method of estimating average life by comparing stub curves with the 18 type survivor curves is remarkably accurate when the many factors are taken into consideration which tend to change the curve from time to time. The simplicity of the method is also a strong recommendation for it.

An alternate method of determining the probable average life of a group of units from a stub survivor curve developed from the experience of the first units to be retired is to extend the curve by eye and judgment. Obviously, the method presented above is much to be preferred for it allows the use of judgment as well as offering the experience of the general law of distribution of retirement followed by all industrial properties.

Wolf and Fitch Reference

Depreciation Systems

$$Y = a_0 + a_1X + a_2X^2 + a_3X^3 + \ldots + a_nX^n$$

Standard regression techniques and computer programs can be used to find the regression coefficients a_i. Although this technique works well for smoothing, the polynomial function should only be used with great care to extrapolate data. In *Statistical Theory with Engineering Applications* Abraham Hald (1952:559) states, "From a purely statistical point of view the regression curve provides a description of the interrelation between the two variables within the limited range of the observations, and extrapolations, i.e., computations or values outside this range are in principle not justifiable as perhaps it is not possible to represent the interrelation outside the observed range by the function utilized. It is therefore absolutely necessary that extrapolation be firmly based on professional knowledge concerning the data." A polynomial curve may not be a good function to use for the difficult task of extrapolation.

If the Iowa curves are adopted as a model, an underlying assumption is that the process describing the retirement pattern is one of the 22 processes described by the Iowa curves. The problem is then to decide which specific type of Iowa curve "best" fits the observed data. *Best* can take on different meanings, each with subtle differences; here it will refer to the curve that most accurately represents the observed data.

One method is to fit the data visually. Until recently, this required a set of curves printed on translucent paper. Printed on each sheet is a family of a specific type Iowa curve. Each member of the family represents a different average life, typically running from 10 to 50 years in steps of 2 years. Traditionally these curves were scaled to 4 years/inch and 10% surviving/inch, but sets of curves scaled to one-half or double this size were also common. These scales can be multiplied or divided by a constant to accommodate observed data with very long or very short lives. If, for example, the observed curve had an average life of about 80 years, the scale could be doubled so that the curves would run from 20 to 100 years. The observed curve was plotted on graph paper using the same scale, and a translucent sheet of paper with the printed curves was then placed over the observed curve, allowing the analyst to compare visually the empirical and observed curves.

After plotting the observed curve, the analyst should first visually examine the plotted data to make an initial judgment about the type curves that may be good fits. The analyst also must decide which points or sections of the curve should be given the most weight. Points at the end of the curve are often based on fewer exposures and may be given less weight than points based on larger samples. The weight placed on those points will depend on the size of the exposures. Often the middle section of the curve (that section ranging from approximately 80% to 20% surviving) is given

more weight than the first and last sections. This middle section is relatively straight and is the portion of the curve that often best characterizes the survivor curve.

Begin fitting with the left modal curves and identify the two or three curves that appear to best fit the data. Note the curve type and the corresponding average life, which is typically estimated to the nearest year. Continue with the symmetrical, right modal, and origin modal curves. Some groups may not give a suitable fit.

Continue by reexamining the contenders selected during the first pass. Often the choice between two or three tentative selections is difficult to make. The conservative choice is toward the lower life and right modal curve.

An alternative to visual fitting is mathematical fitting. Usually the least squares method is used. This method is time consuming if done by hand, and is not practical unless a computer is used. Typical logic for a computer program is as follows. First a type curve is arbitrarily selected. If the observed curve goes to zero percent surviving, calculate the area under the curve and designate this the average life.

If the observed curve is a stub curve (i.e., if it does not go to zero), calculate the area under the curve and up to the age at final data point. Call this area the *realized life*. Then systematically vary the average life of the theoretical survivor curve and calculate its realized life at the age corresponding to the study date. This trial and error procedure ends when you find an average life such that the realized life of the theoretical curve equals the realized life of the observed curve. Call this the *average life*.

Once the average life is found, calculate the difference between each percent surviving point on the observed survivor curve and the corresponding point on the Iowa curve. Square each difference and sum them. The sum of squares is used as a measure of goodness of fit for that particular Iowa type curve. This procedure is repeated for the remaining 21 Iowa type curves. The "best fit" is declared to be the type of curve that minimizes the sum of differences squared.

On the surface, the removal of judgment from the fitting process may appear to be an advantage, but blind acceptance of mechanical fitting processes will occasionally but consistently result in poor results. A better procedure is to use the least squares method to select candidates for the best fit. Comparison of the sum of squares will reveal situations where the difference between the best choices is small. The analyst should then visually examine the observed data and compare them to the theoretical curves. This can be done quickly on a computer with graphic capabilities so that the analyst need not use time to plot the observed curve by hand. The analyst can consider single points that may contribute significantly to the sum of squares but that may deserve less weight than other points. Fits at